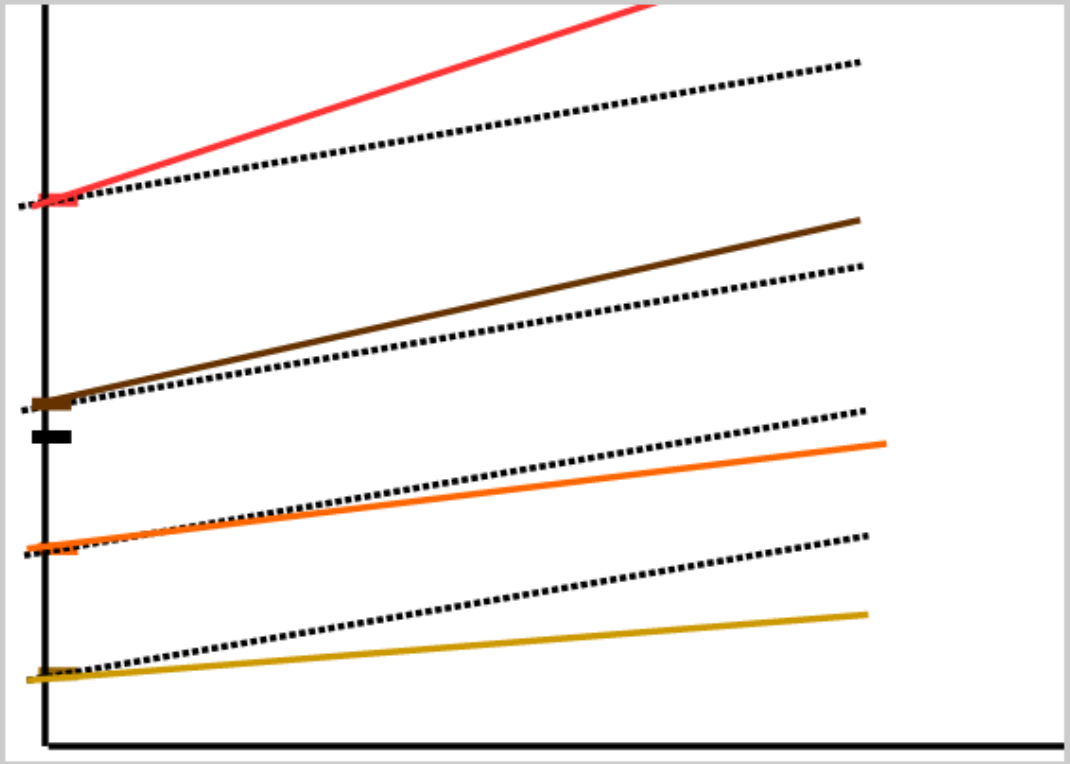


Formeln graphisch:

Mehrebenen-analyse

Arndt Regorz
Dipl. Kaufmann &
B.Sc. Psychologie

Stand:
05.01.2019

Mehrebenenanalyse

- Regression mit Prädiktoren auf mehreren Ebenen
- Kriterium i.d.R. auf unterster Ebene

Ebenenstruktur z.B.

- L1 Arbeitnehmer, L2 Team
- L1 Schüler, L2 Klasse
- L1 Messzeitpunkt, L2 Person

Beispiel

Vorhersage Deutschpunkte (DE)

Prädiktor L1: IQ des Schülers

Prädiktor L2: Unterricht durch Fachlehrer (FL: ja=1, nein=0)

gewöhnliche Regression:

$$Y_i = \beta_0 + \beta_1 X_{1i} + e_i$$

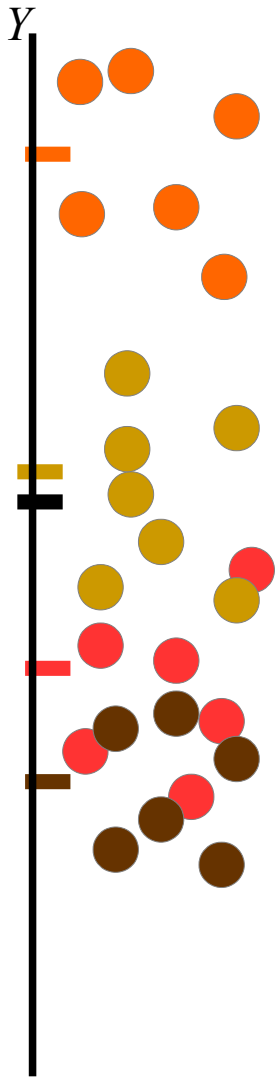
Mehrebenen-Regression:

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \quad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + u_{0j} \quad \text{(L2)}$$

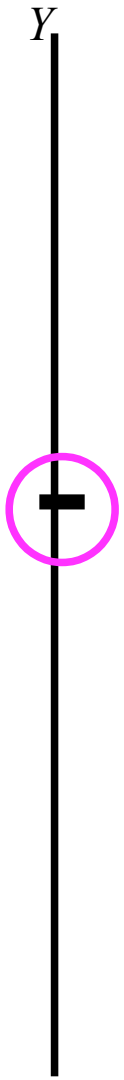
$$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + u_{1j} \quad \text{(L2)}$$

1. Intercept only



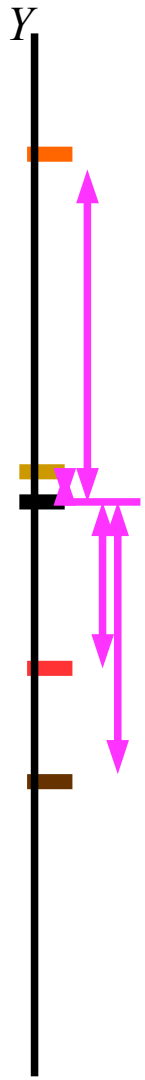
$$Y_{ij} = \beta_{0j} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$



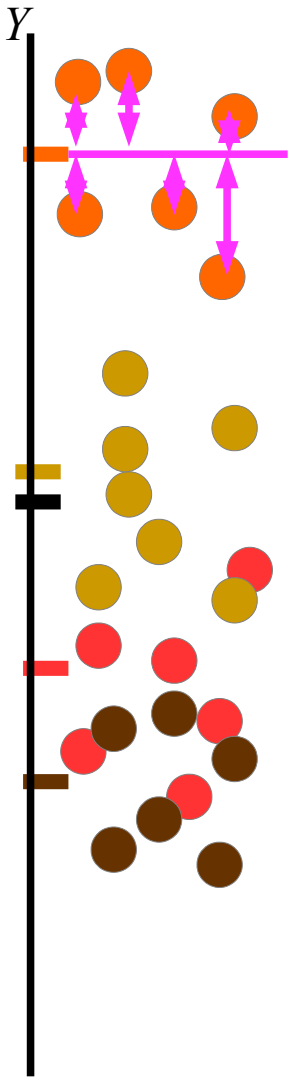
$$Y_{ij} = \beta_{0j} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + e_{ij} \qquad \text{(L1)}$$

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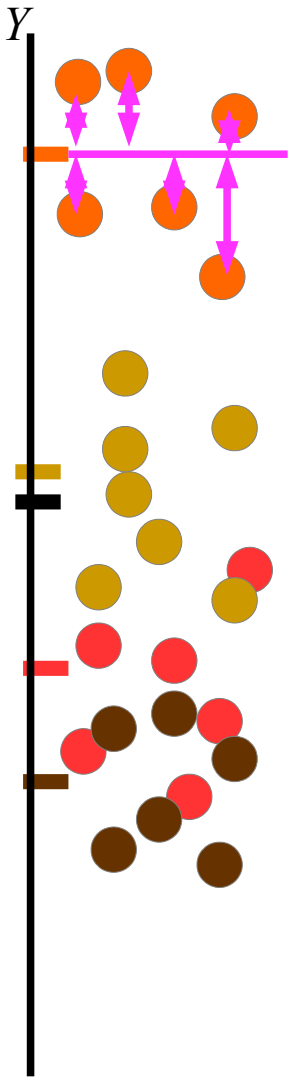
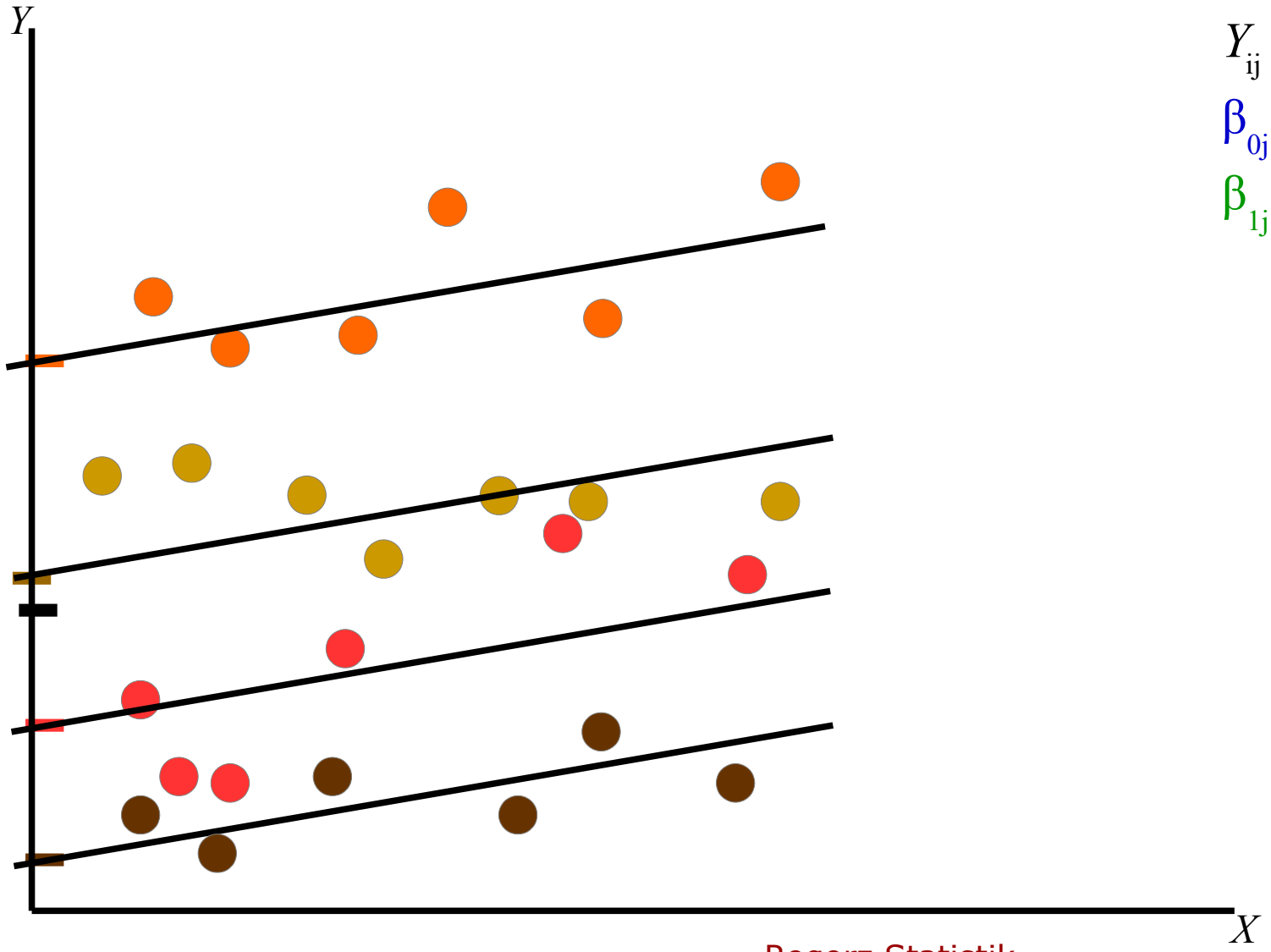


$$Y_{ij} = \beta_{0j} + e_{ij} \quad (L1)$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (L2)$$

Aus diesen beiden Varianzen errechnet man die Intraklassen-korrelation ICC

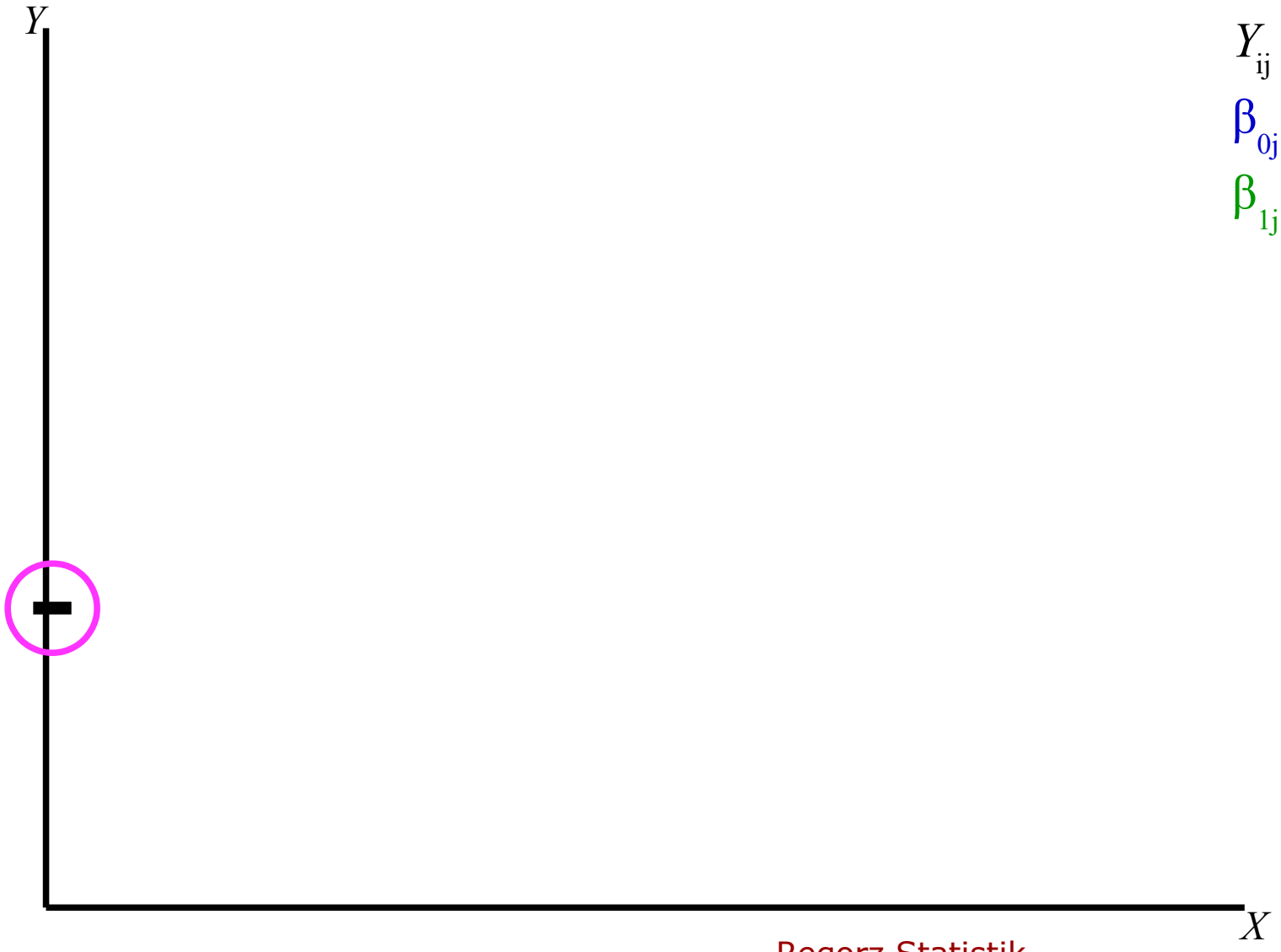
2. + Level 1 Prädiktor



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$

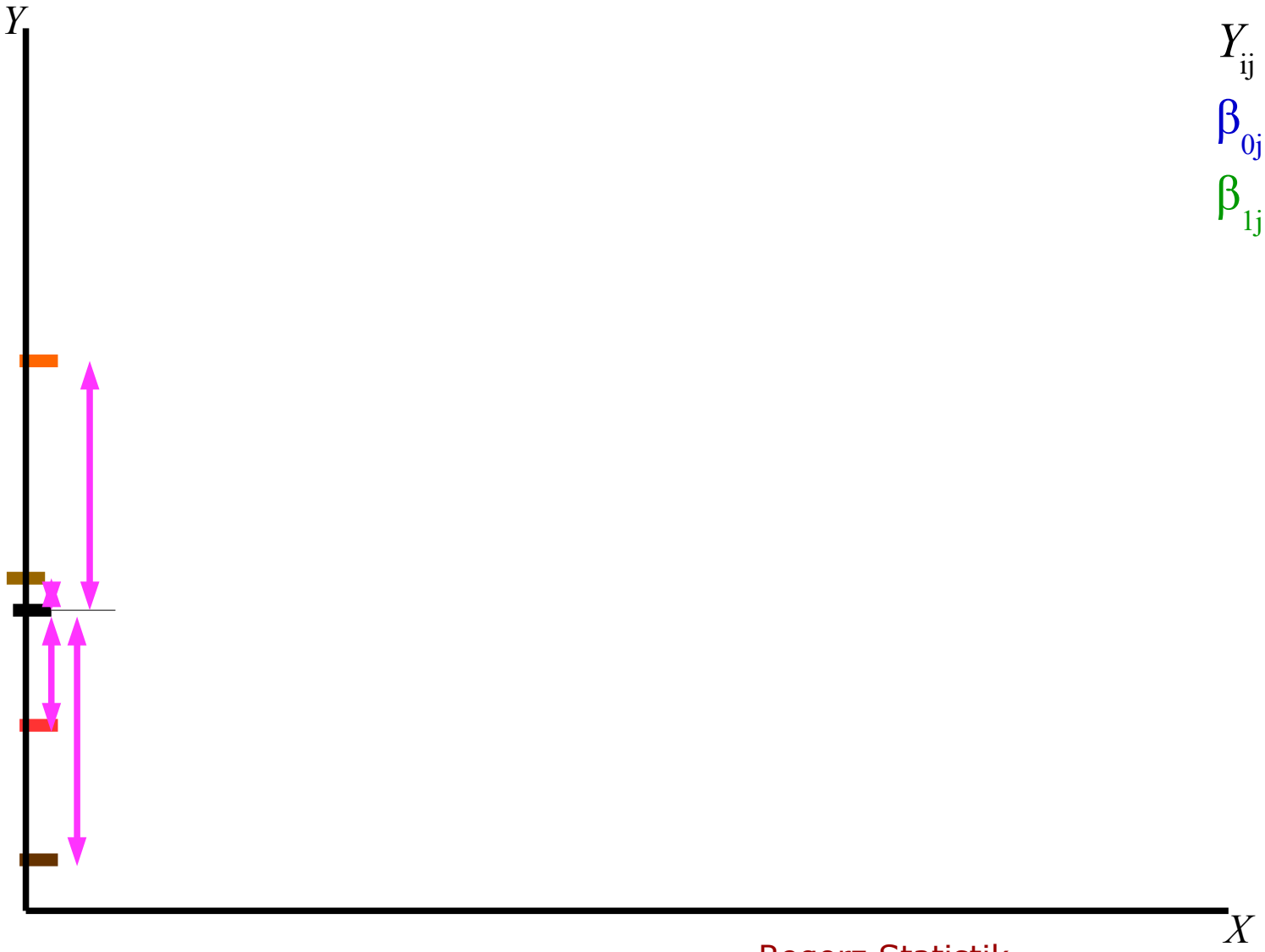
$$\beta_{1j} = \gamma_{10} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$

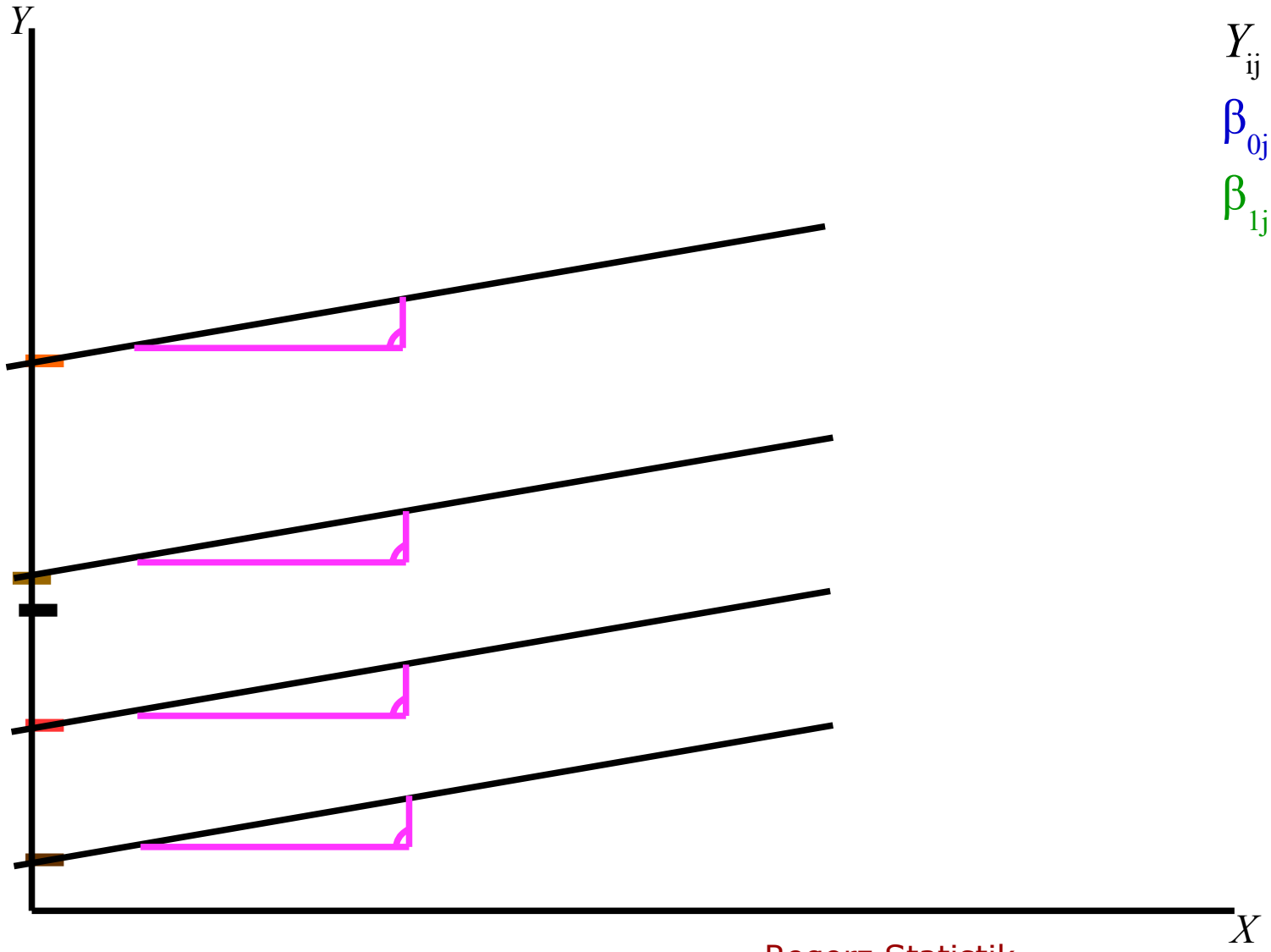
$$\beta_{1j} = \gamma_{10} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$

$$\beta_{1j} = \gamma_{10} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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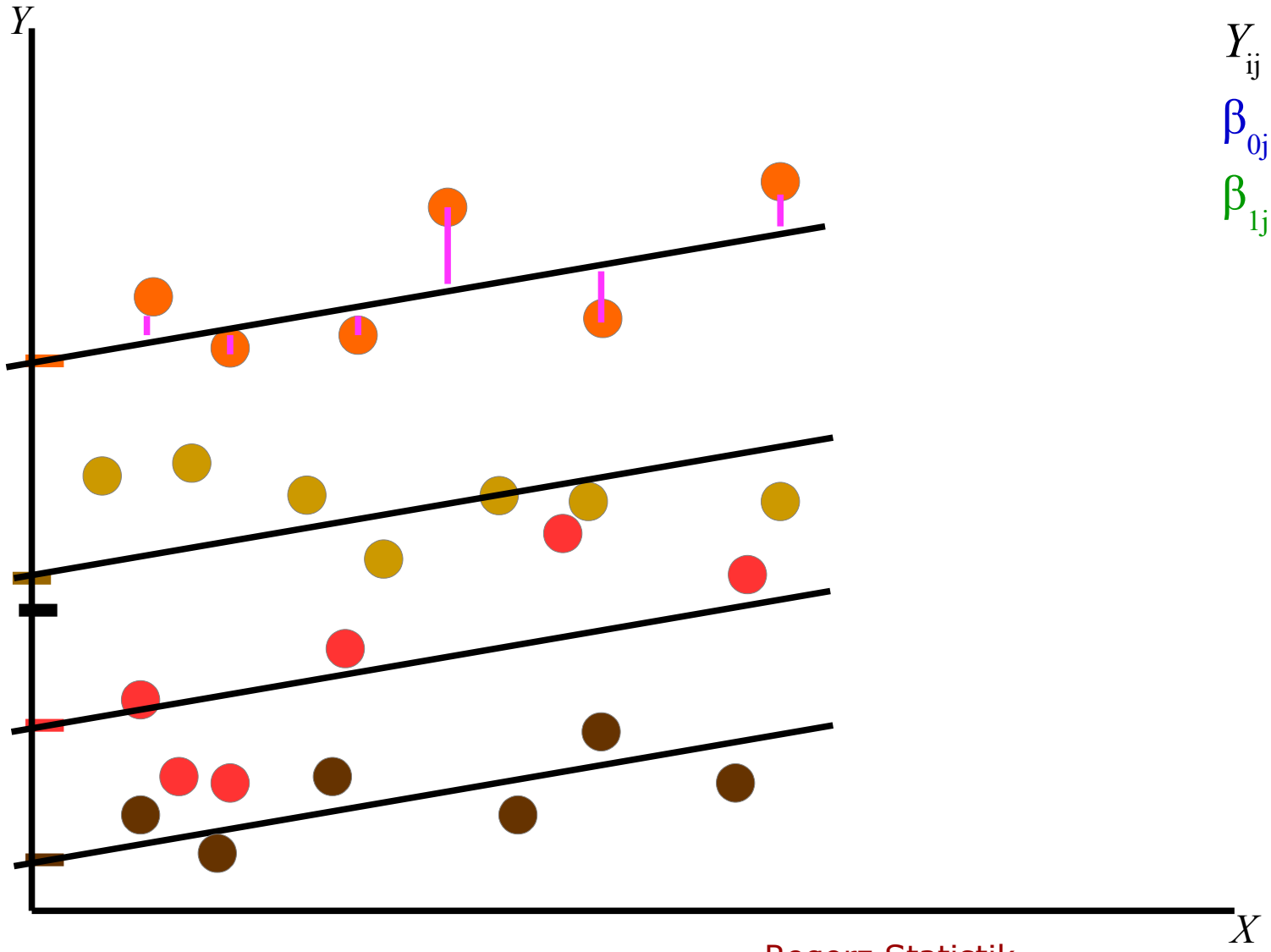
$$\beta_{1j} = \gamma_{10} \qquad \text{(L2)}$$

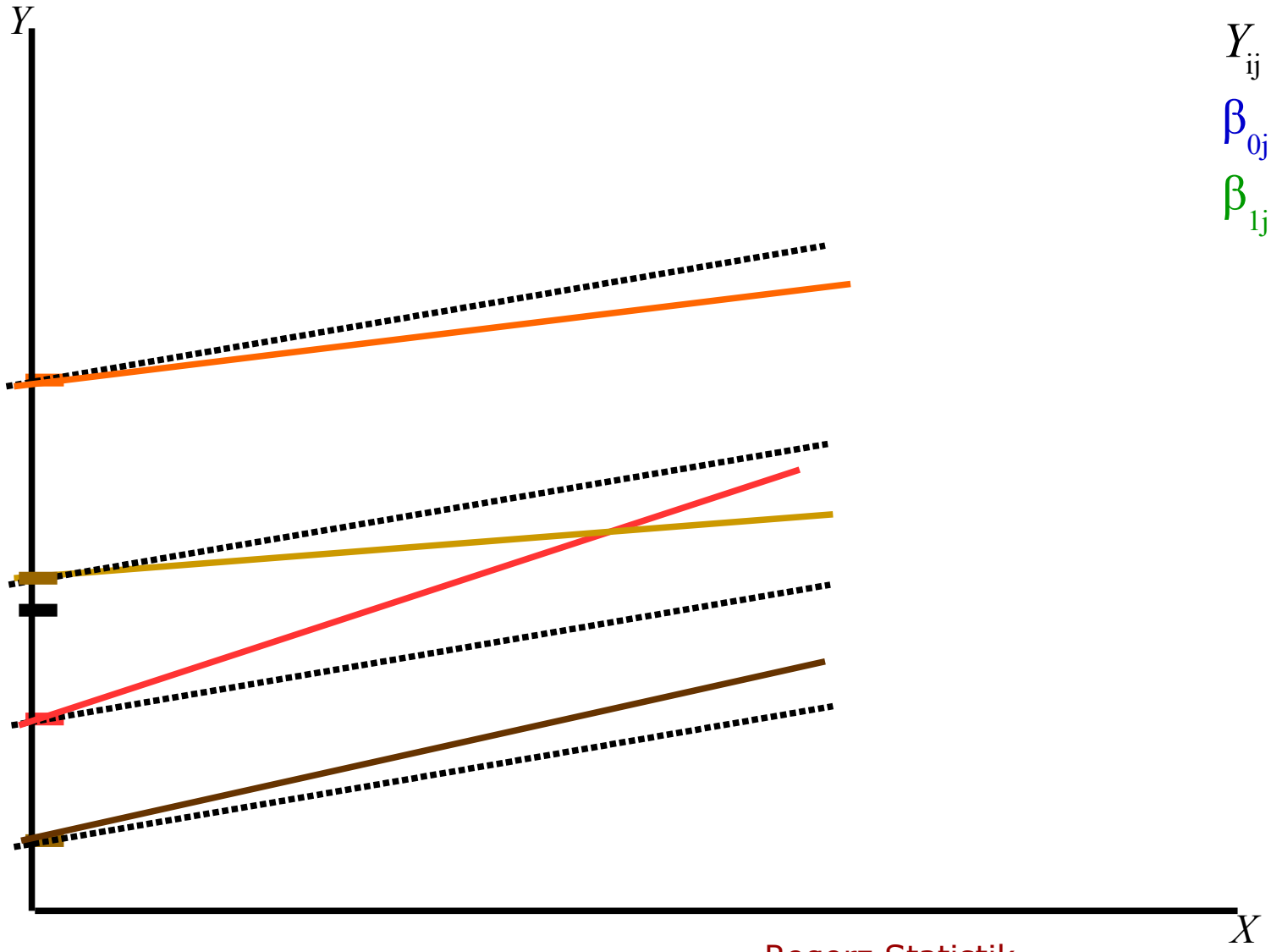


$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$

$$\beta_{1j} = \gamma_{10} \qquad \text{(L2)}$$

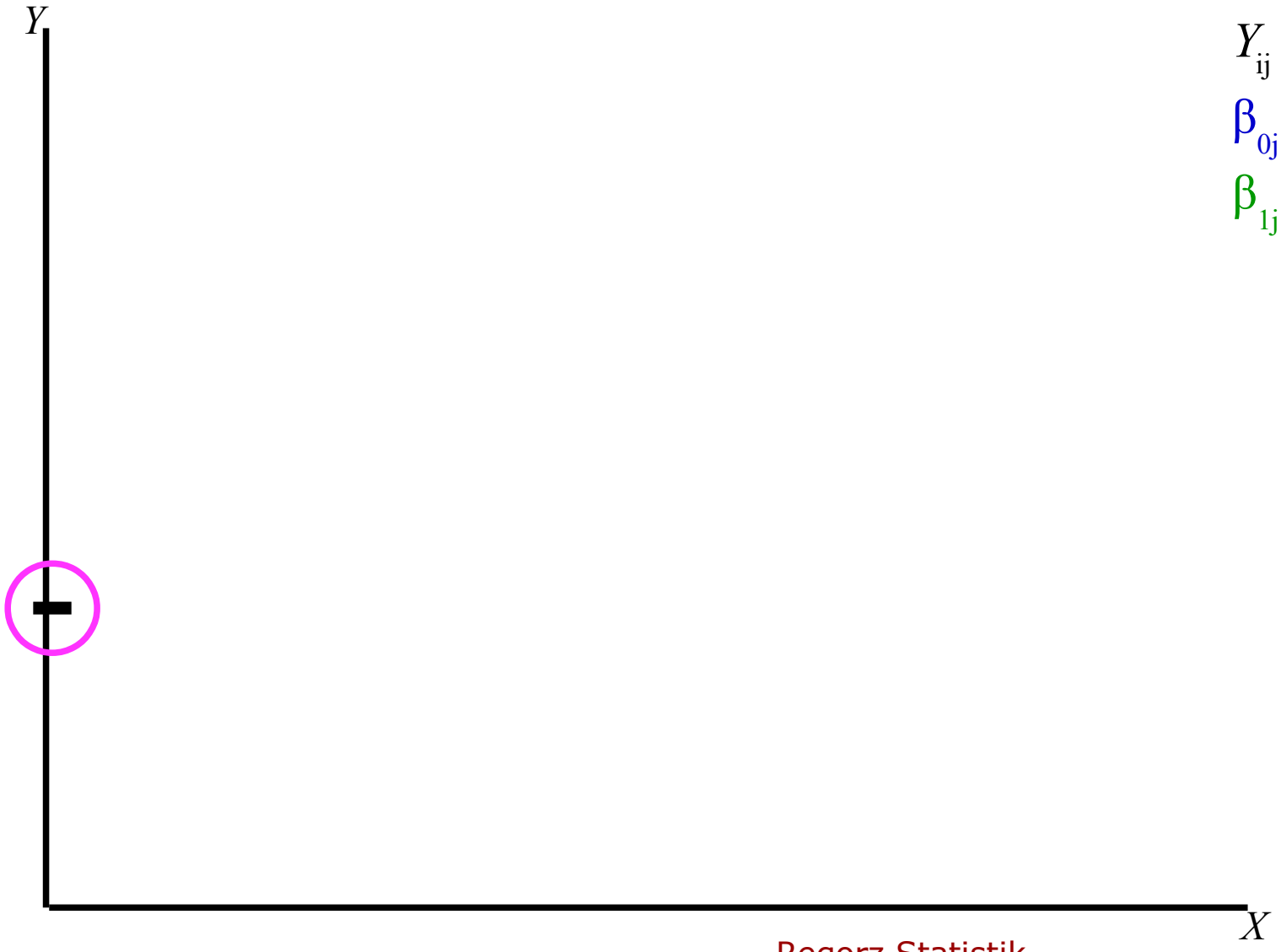
3. + Random Slopes



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \qquad \text{(L2)}$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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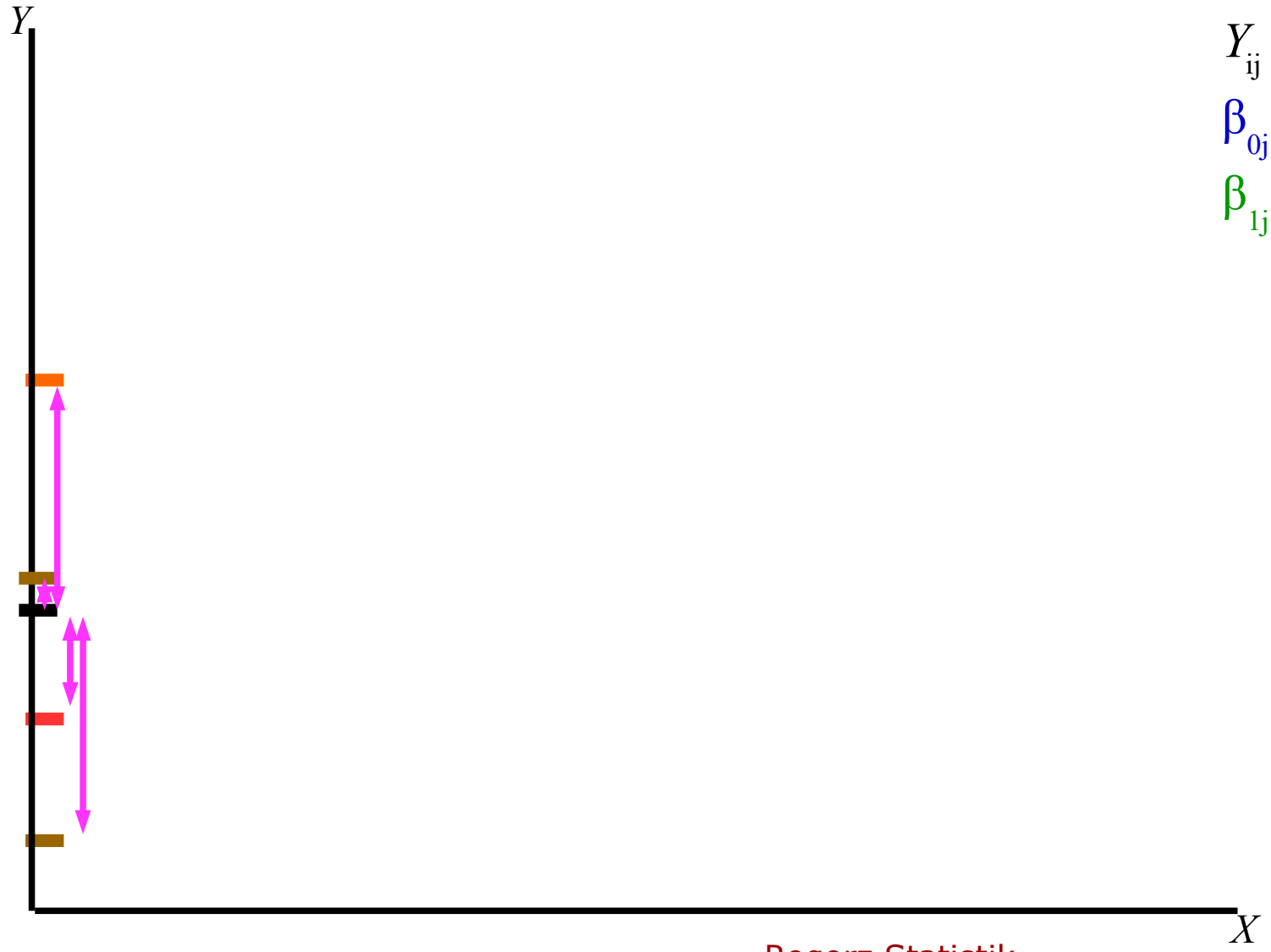

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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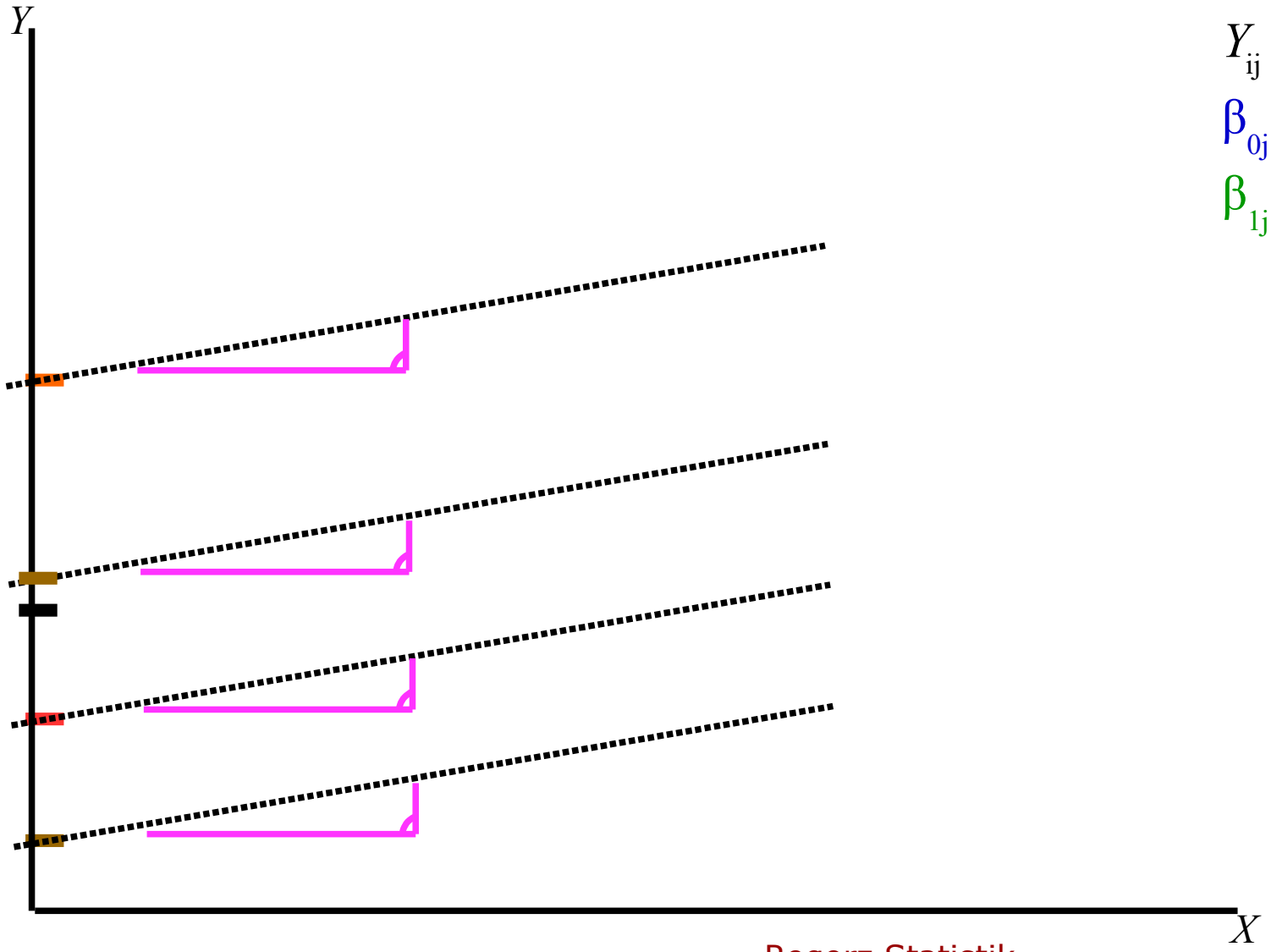

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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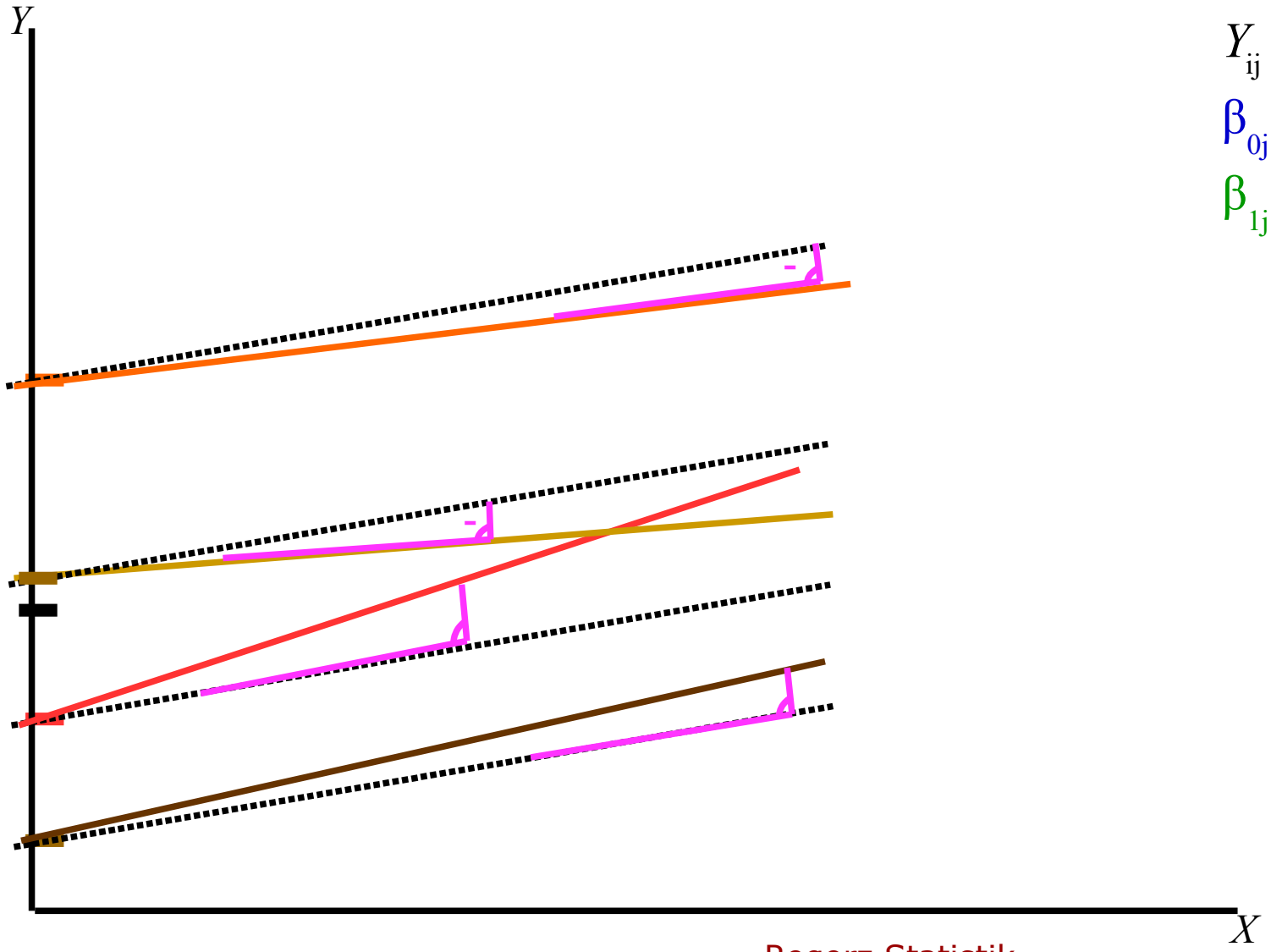
$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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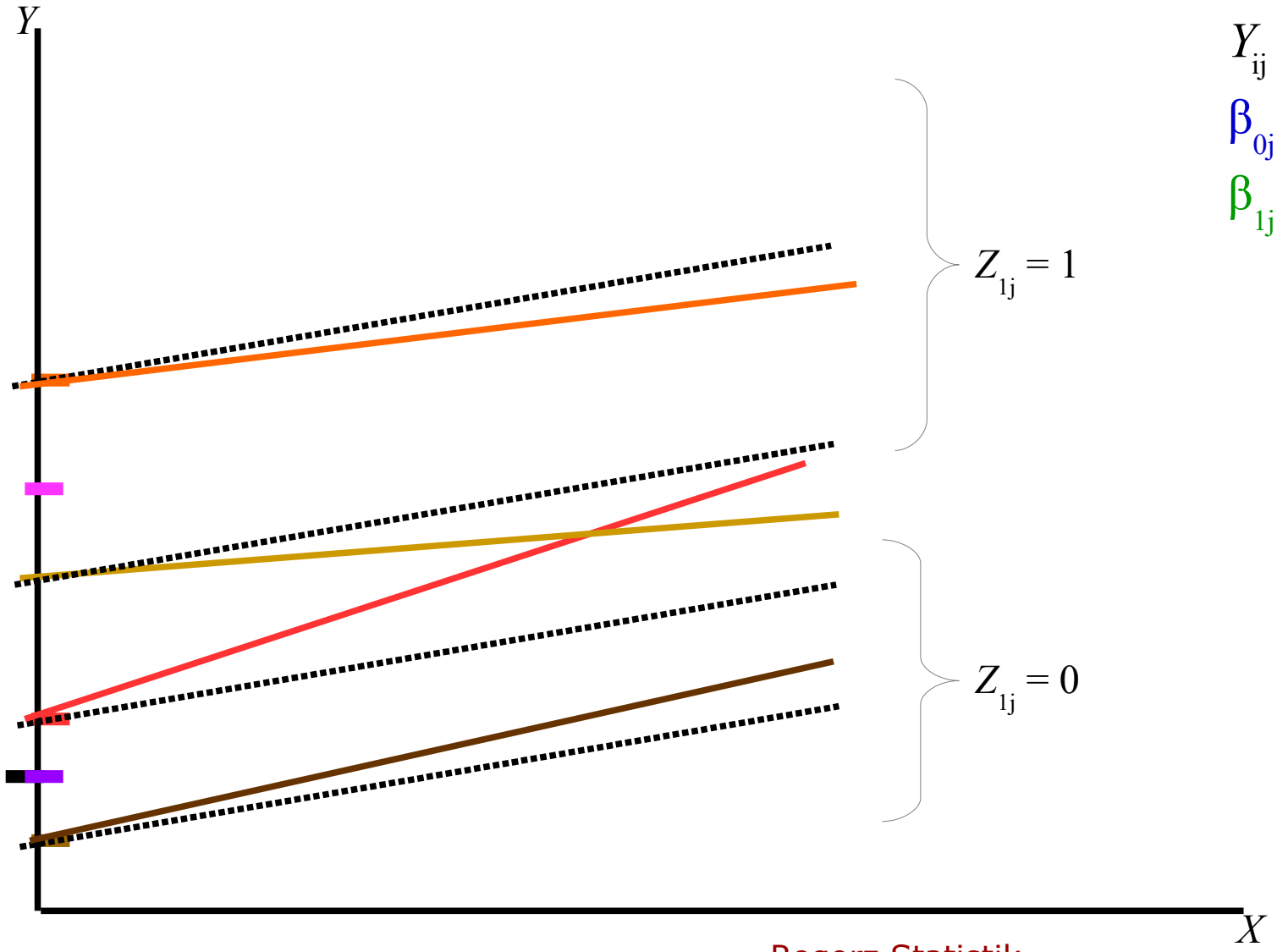
$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$

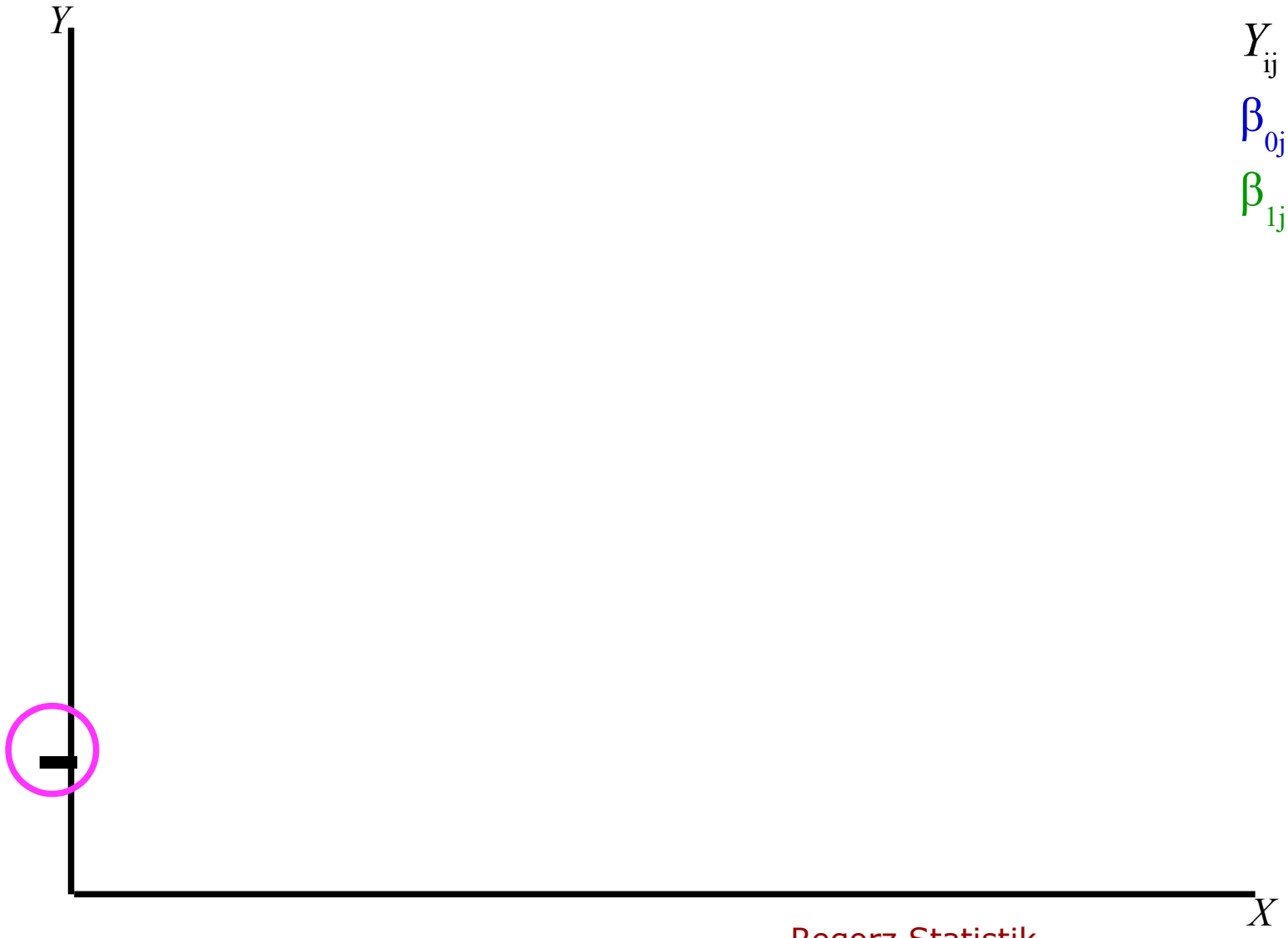
4. + Level 2 Prädiktor



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + u_{0j} \qquad \text{(L2)}$$

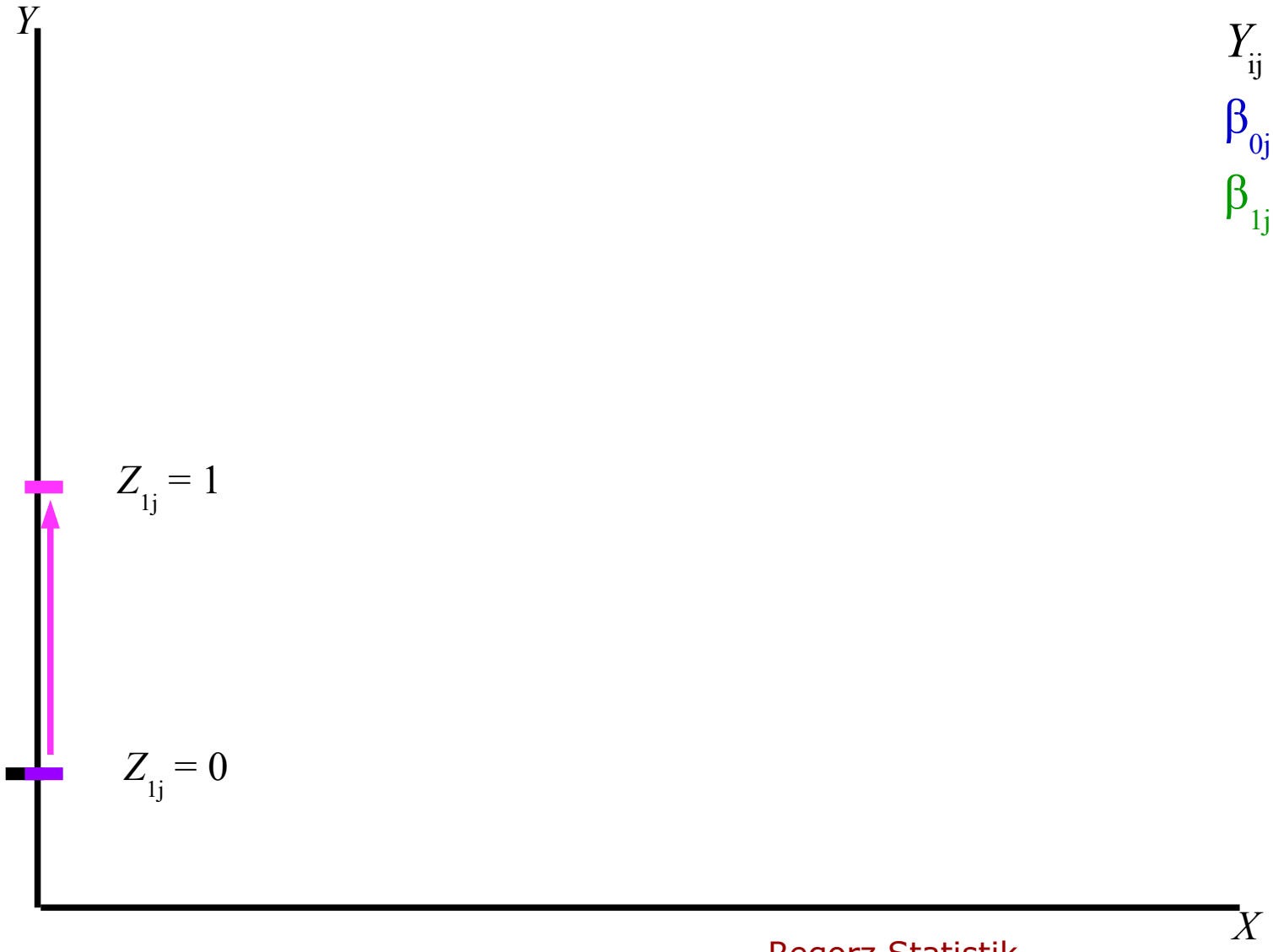
$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + u_{0j} \qquad \text{(L2)}$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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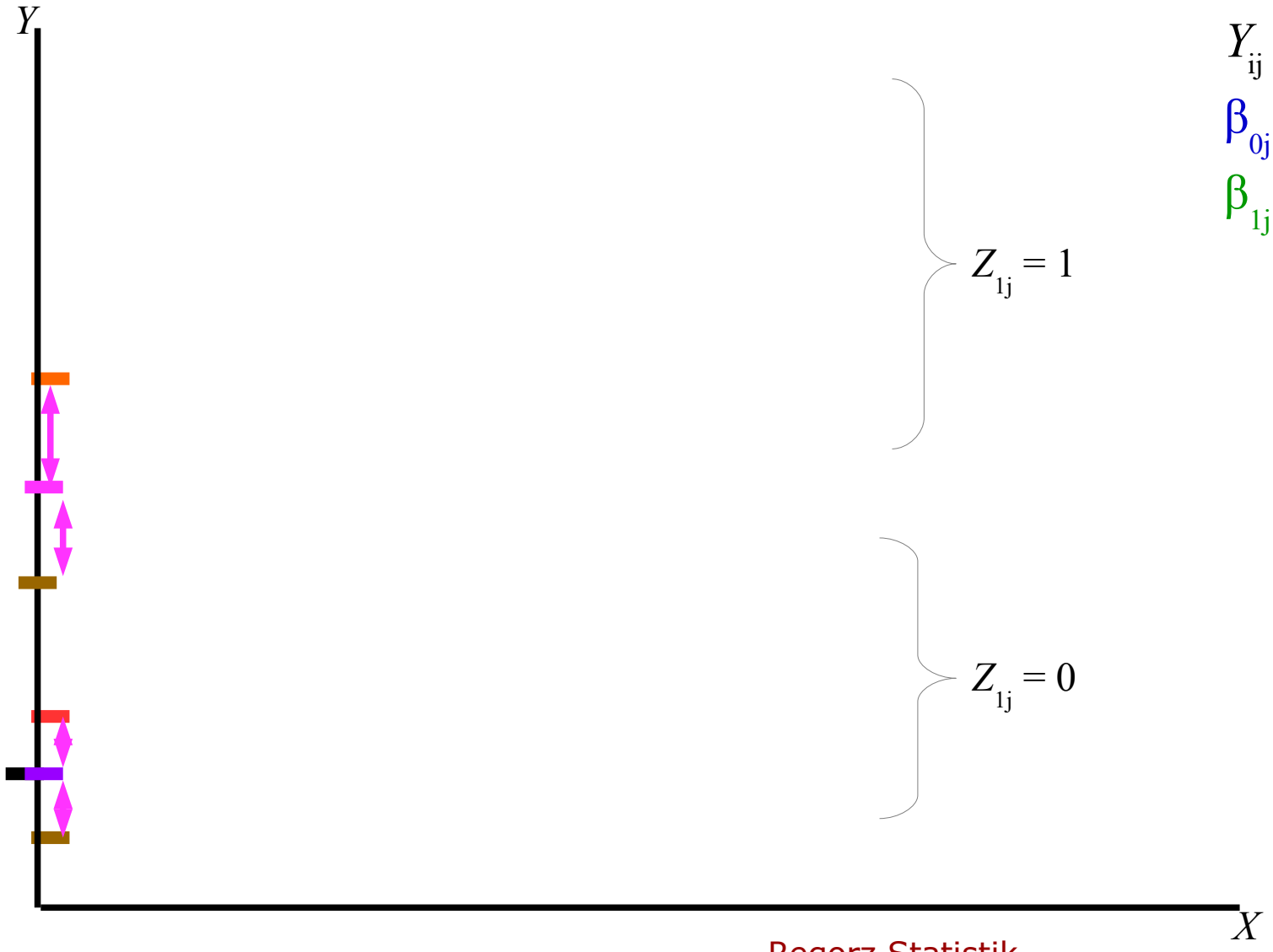

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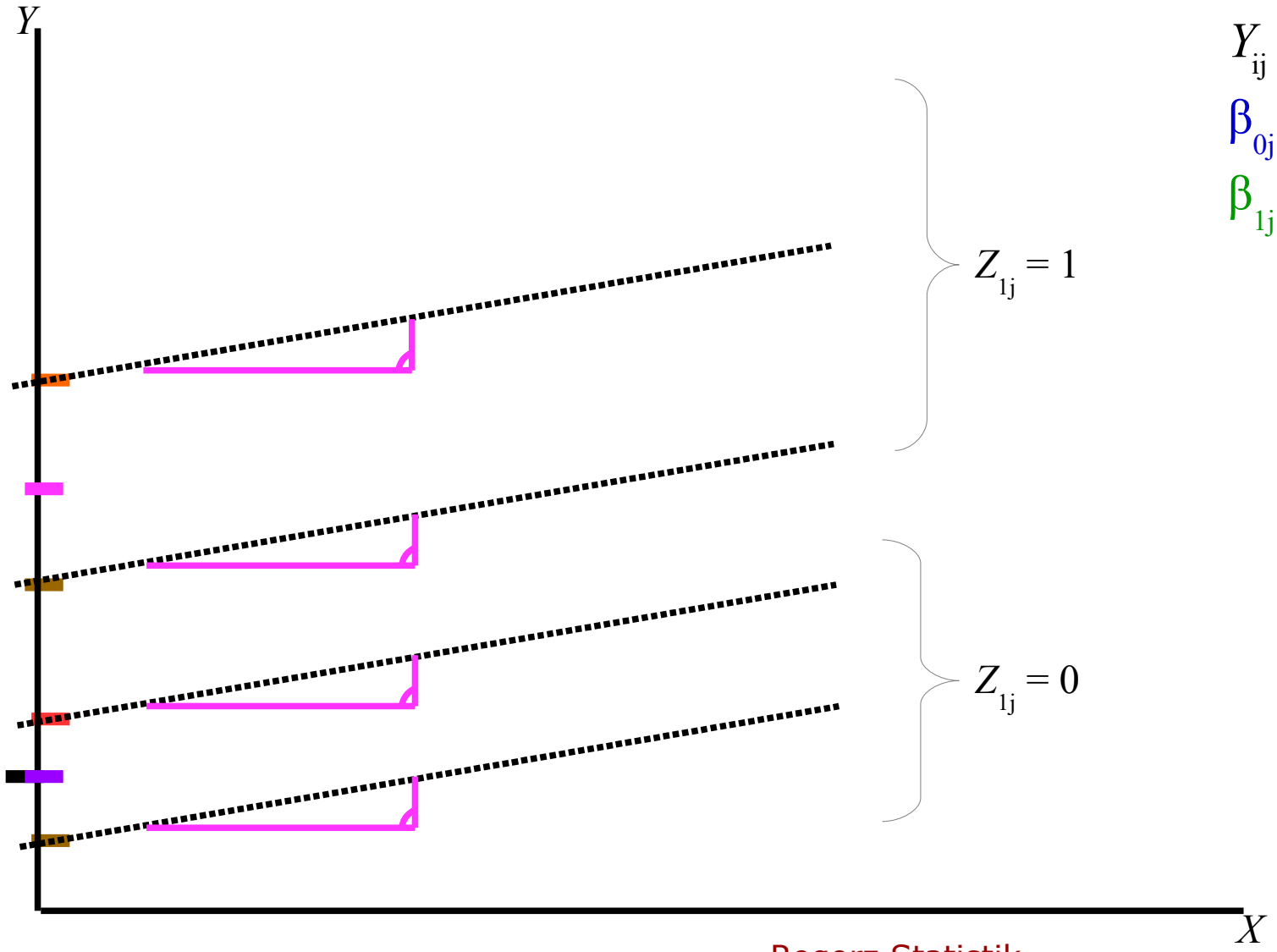

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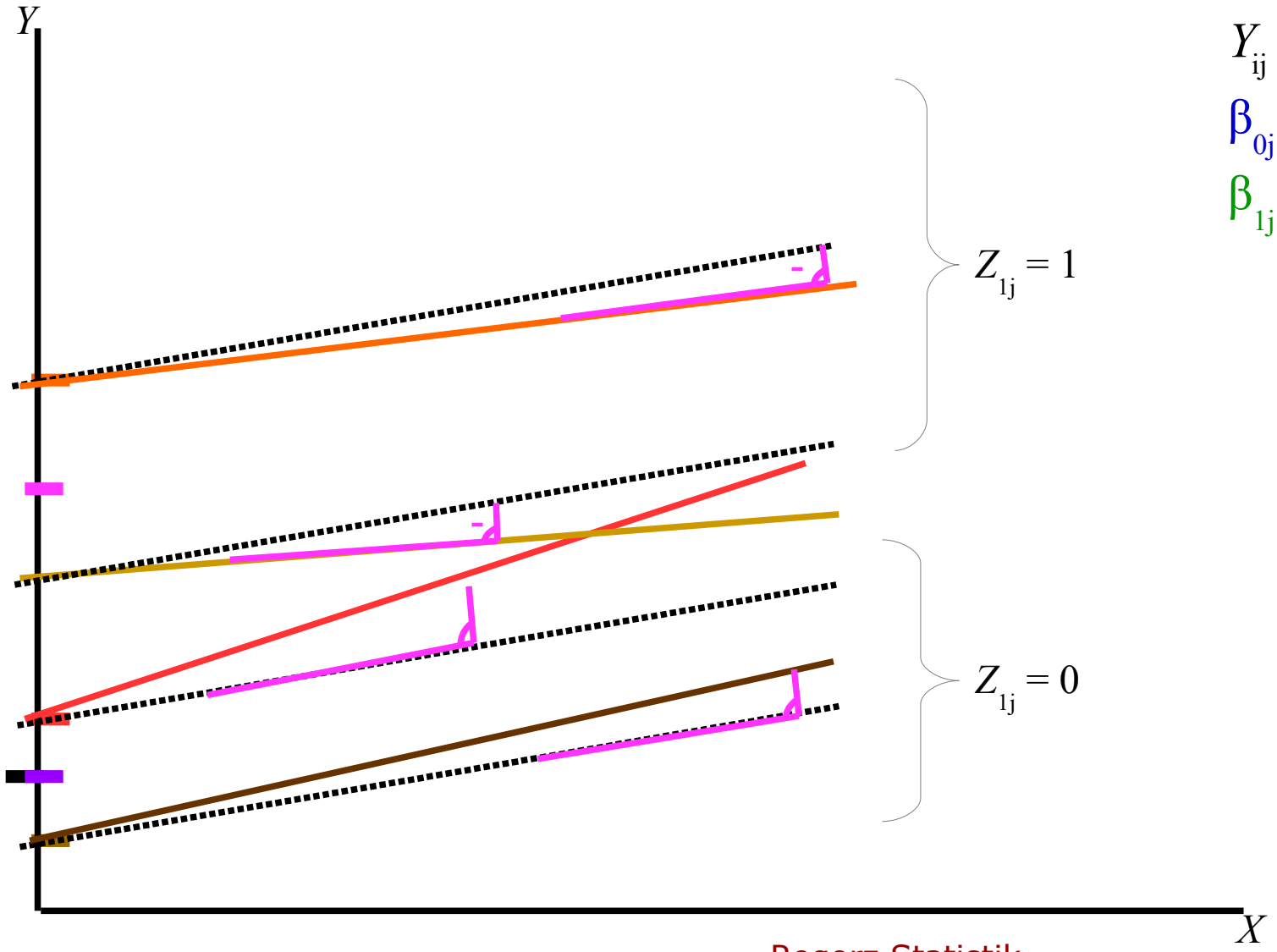

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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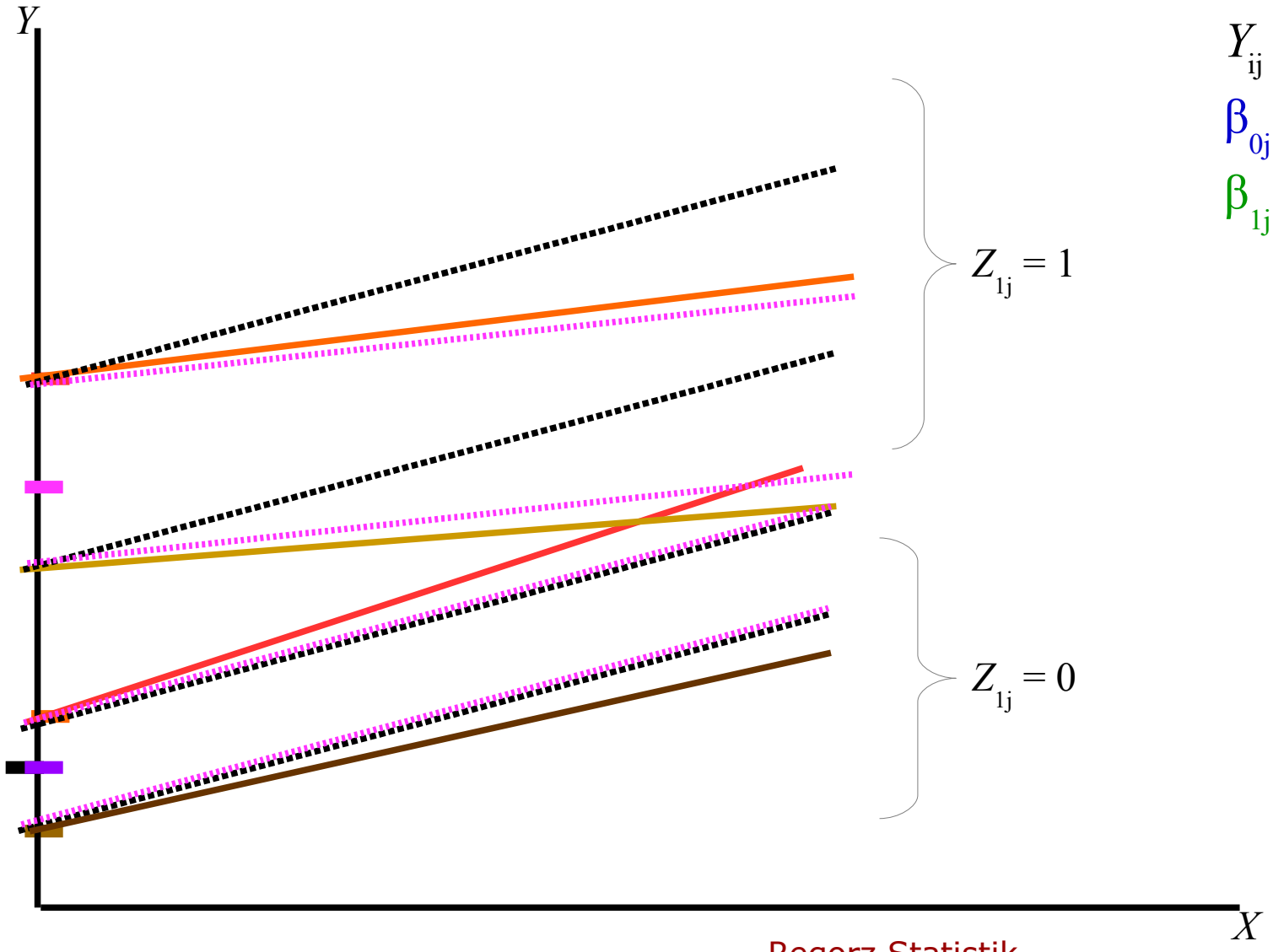
$$\beta_{1j} = \gamma_{10} + u_{1j} \qquad \text{(L2)}$$

5. + Cross-Level-Interaction



$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + e_{ij} \qquad \text{(L1)}$$

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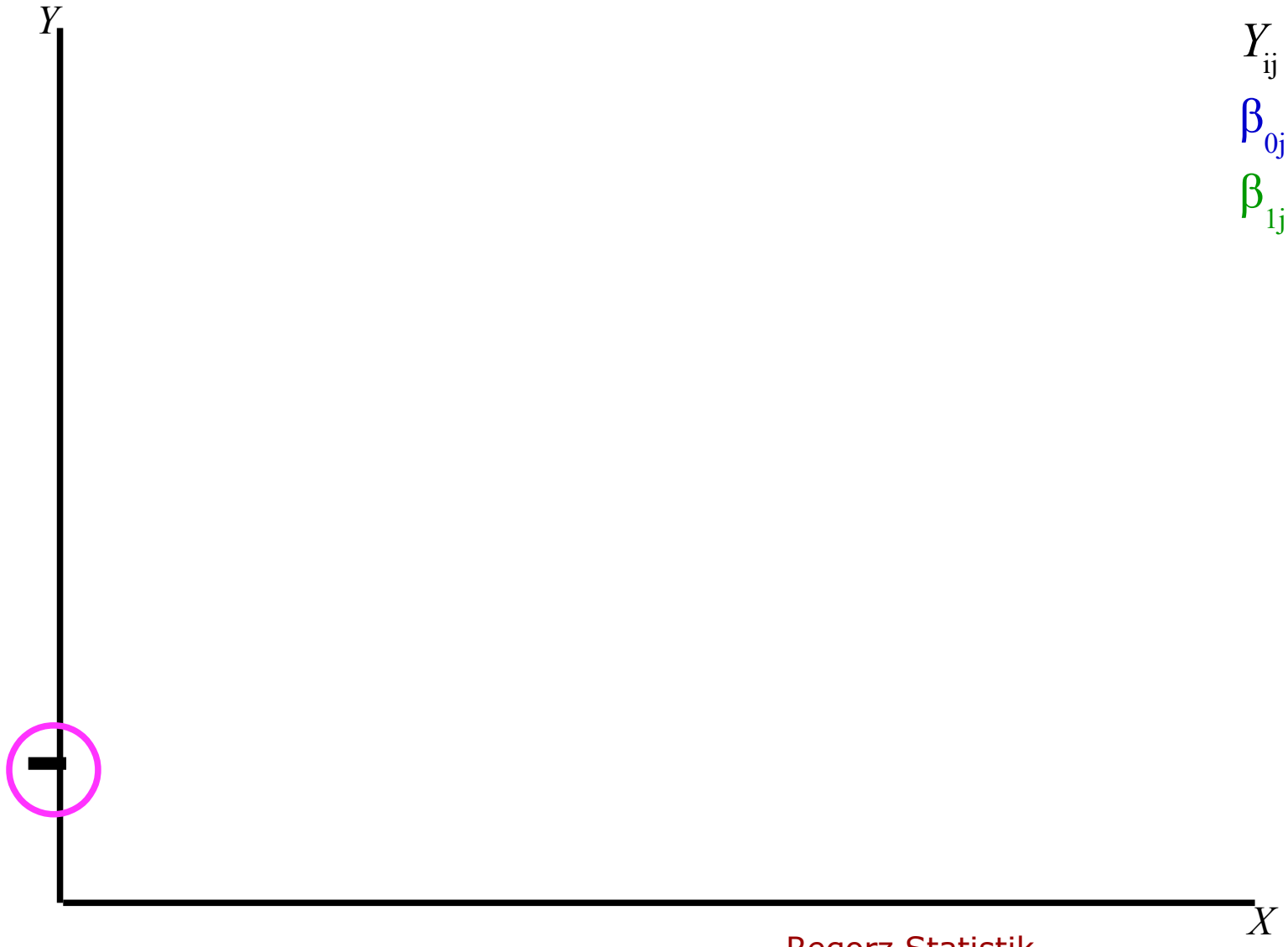

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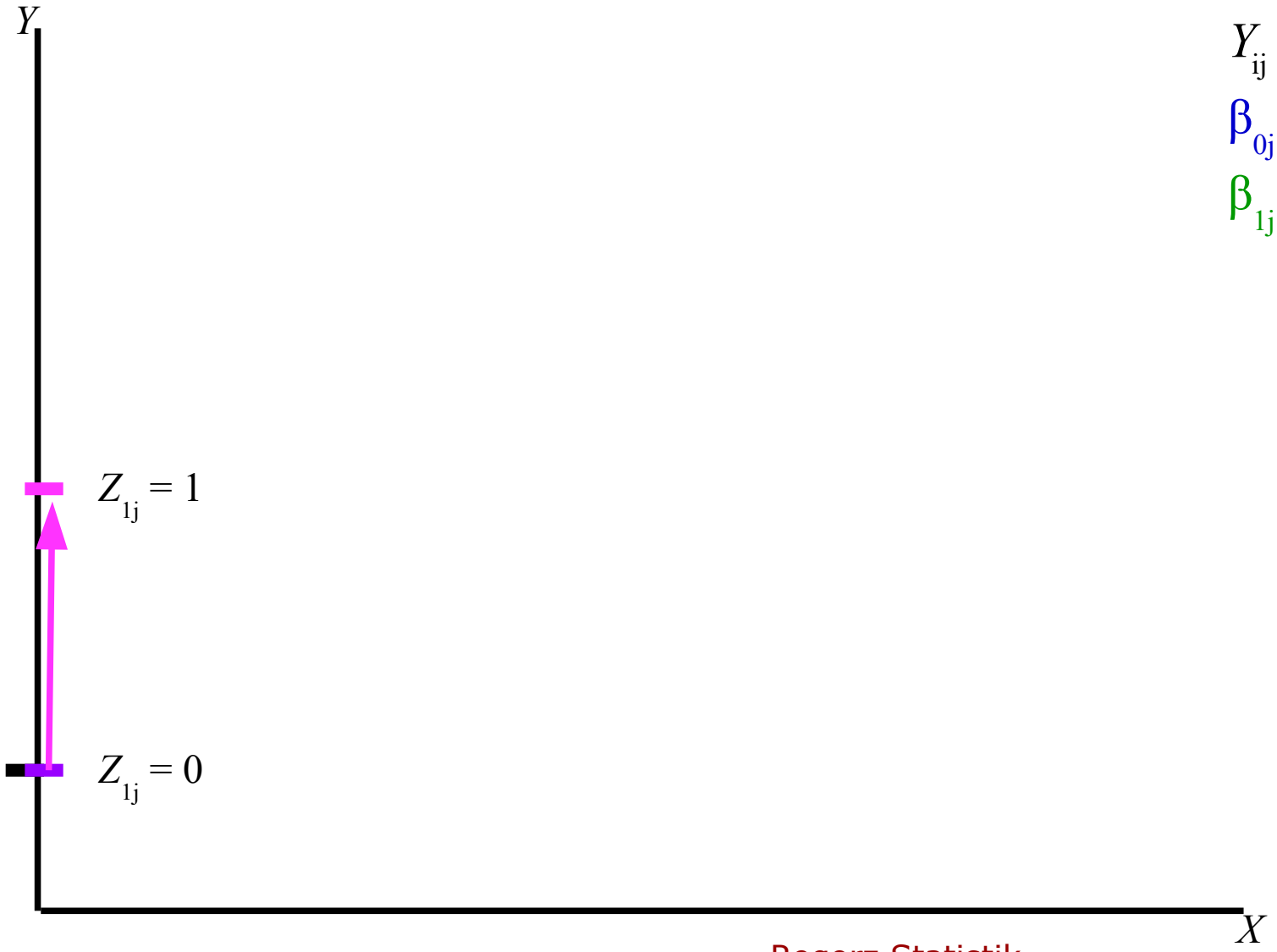

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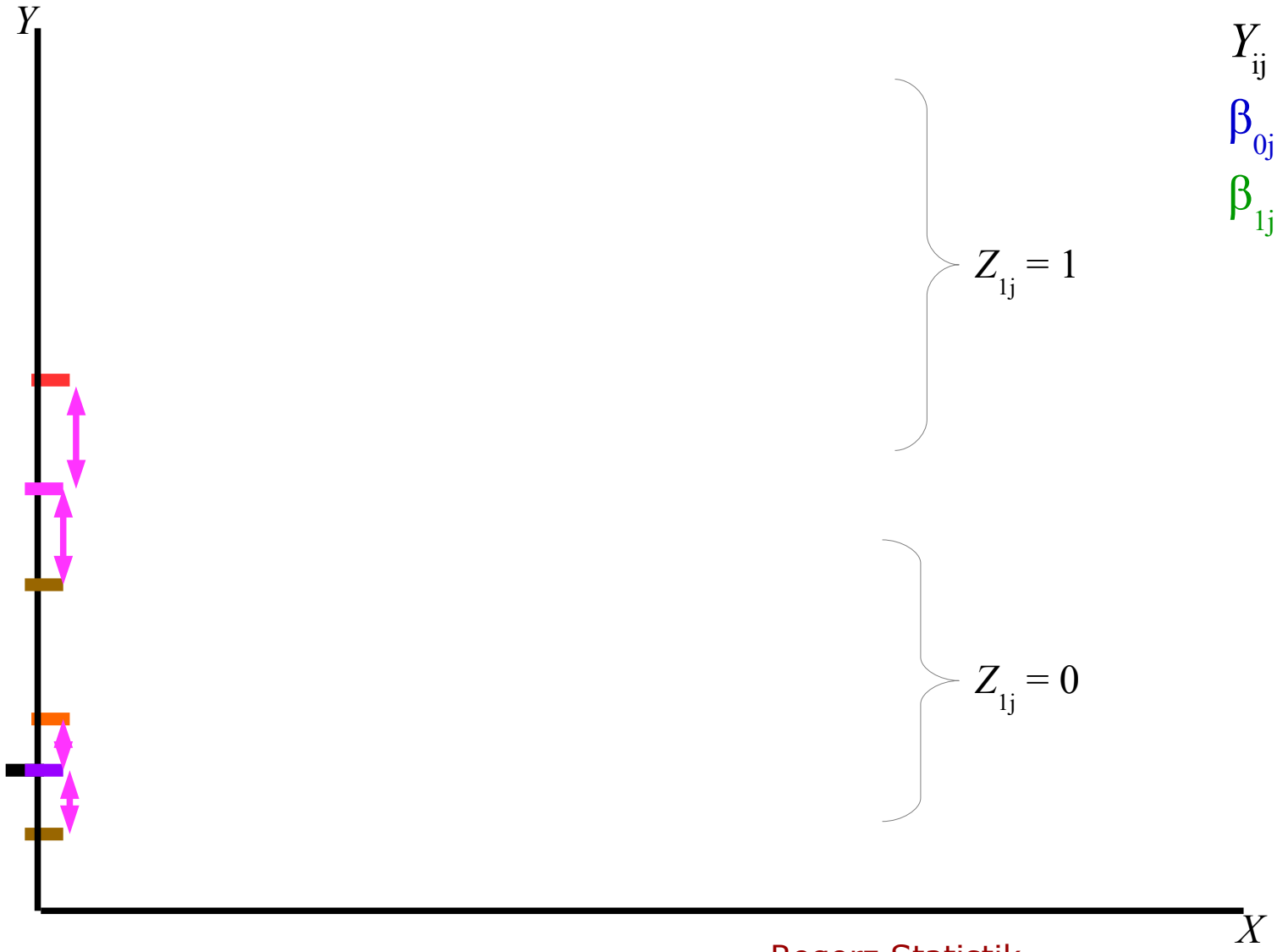

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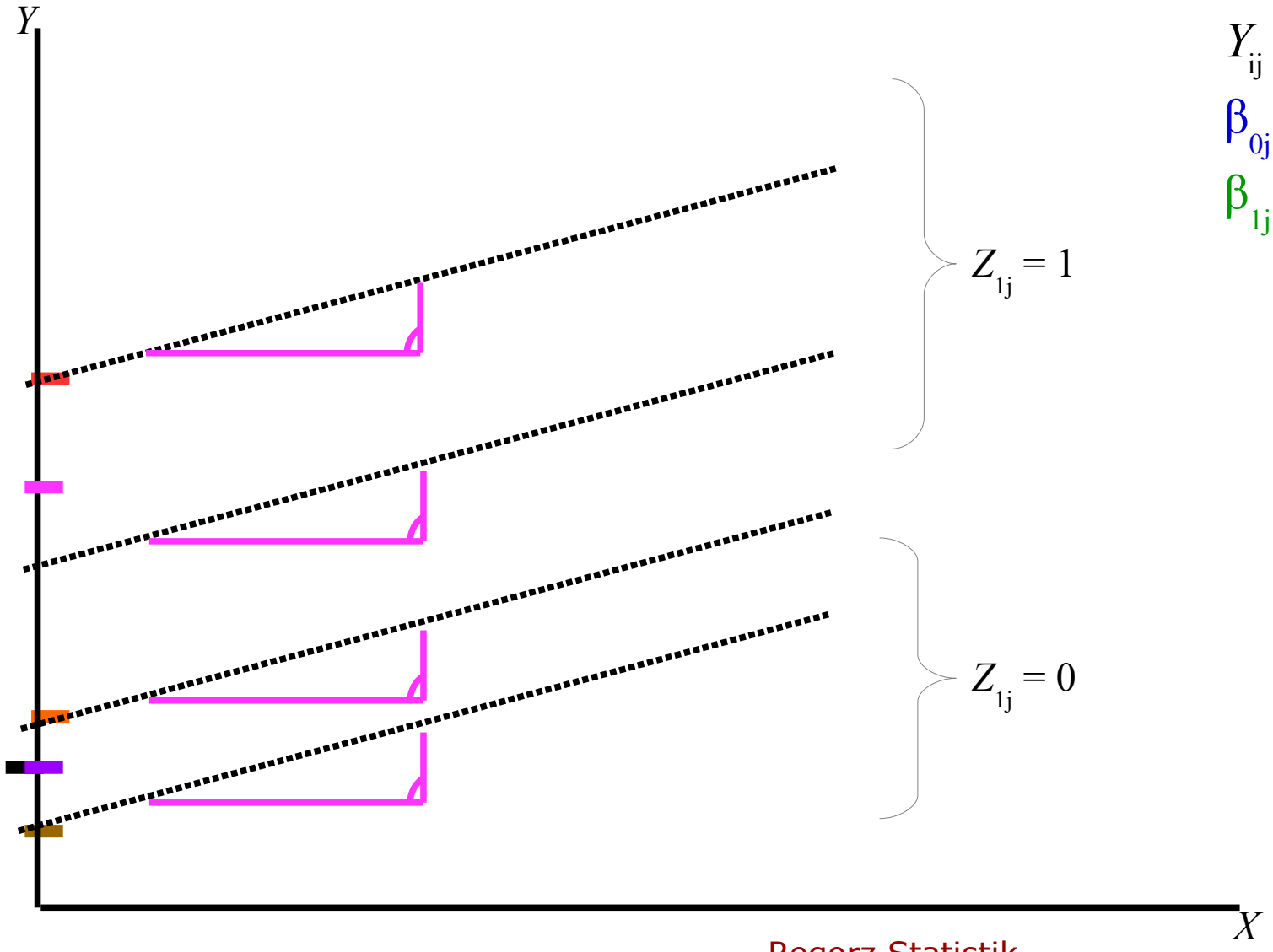

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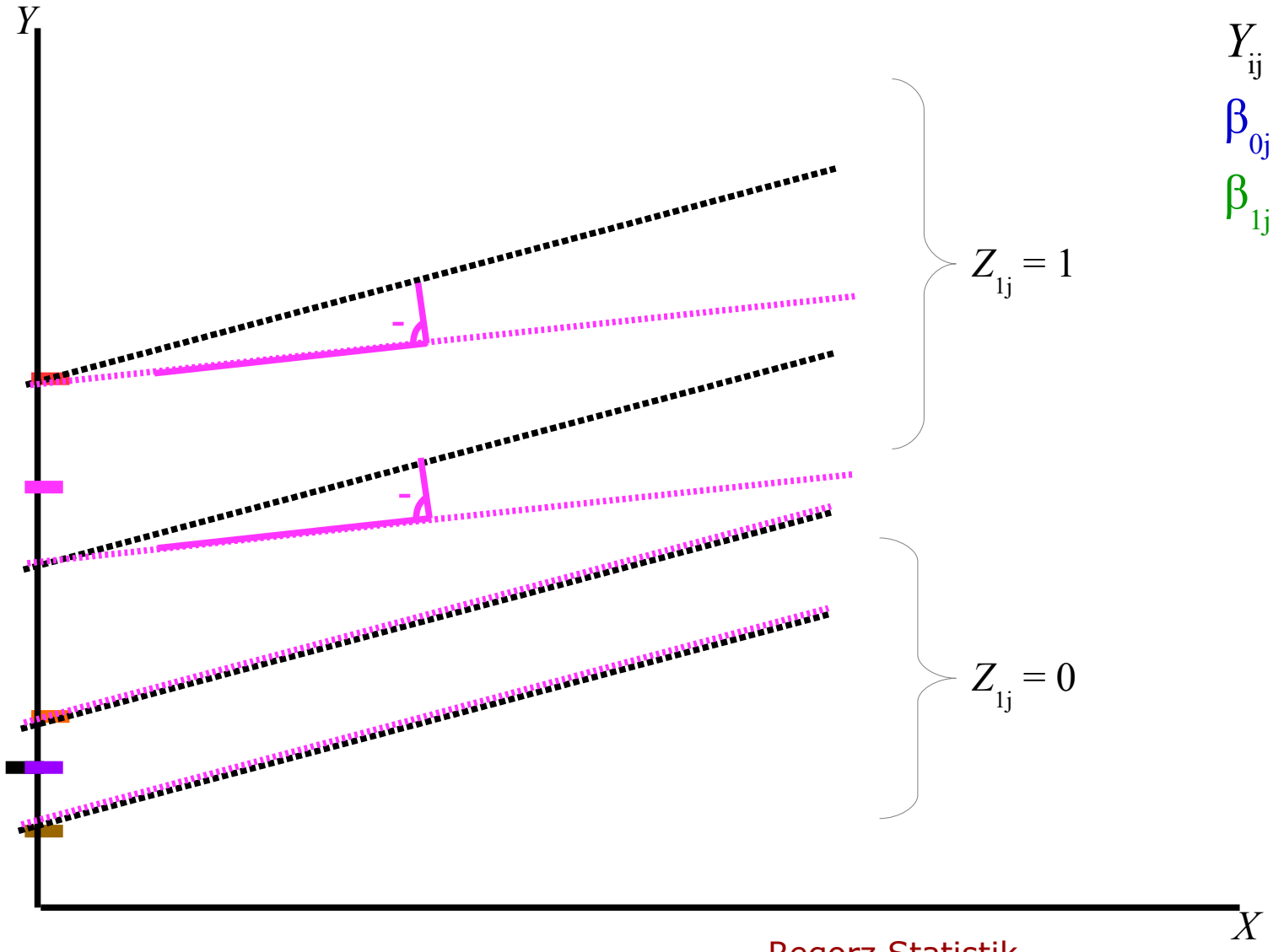

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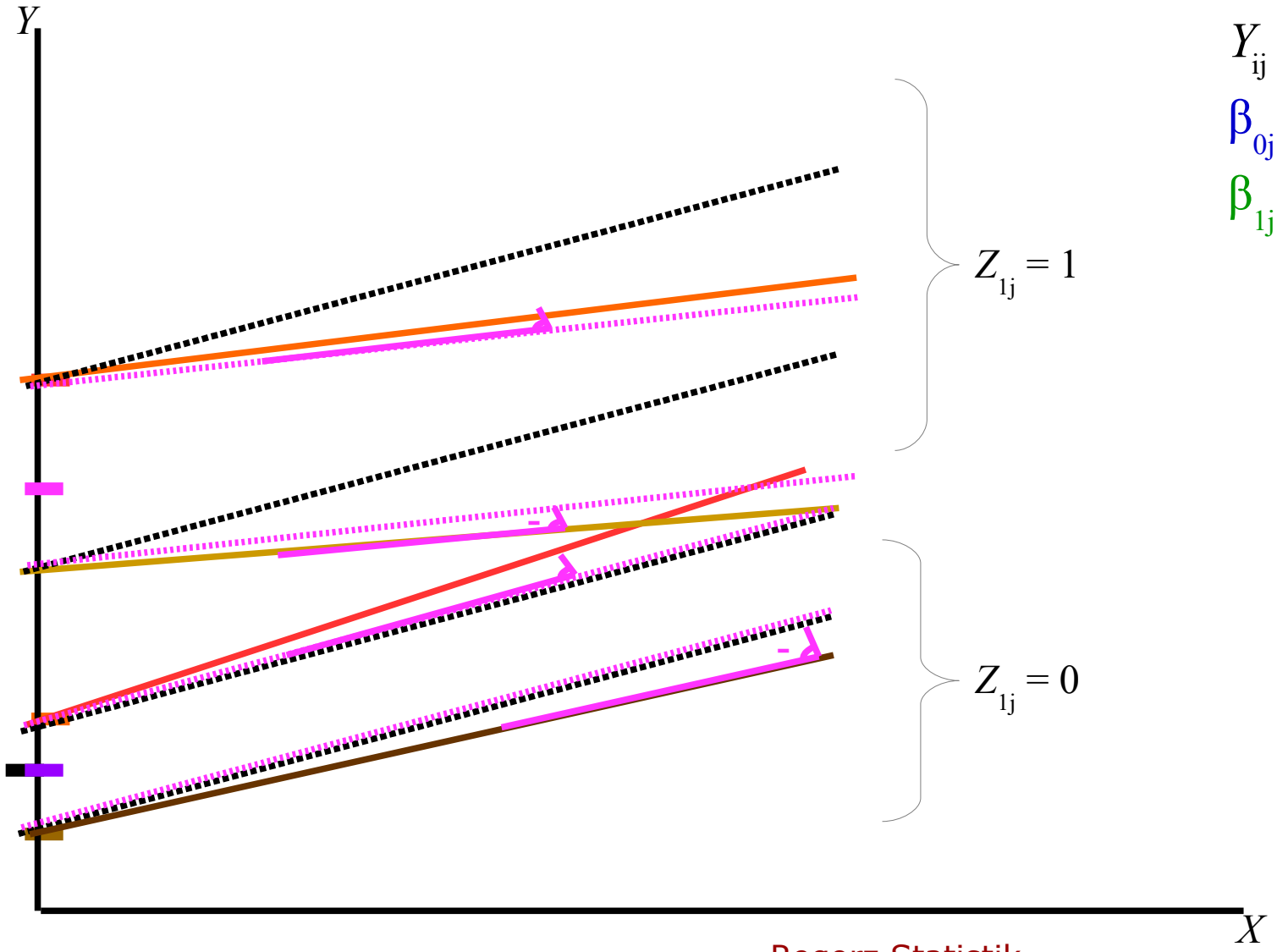

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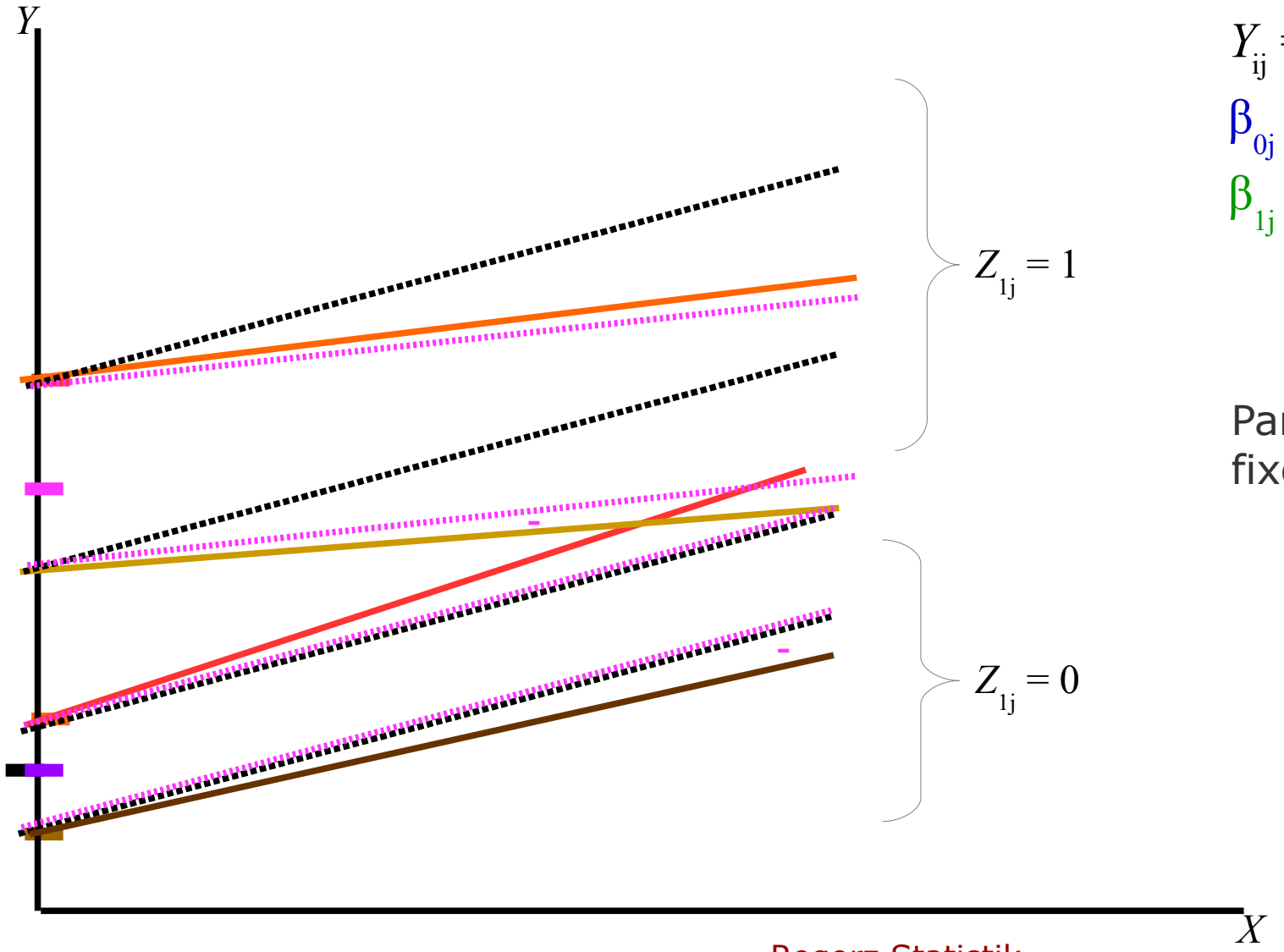

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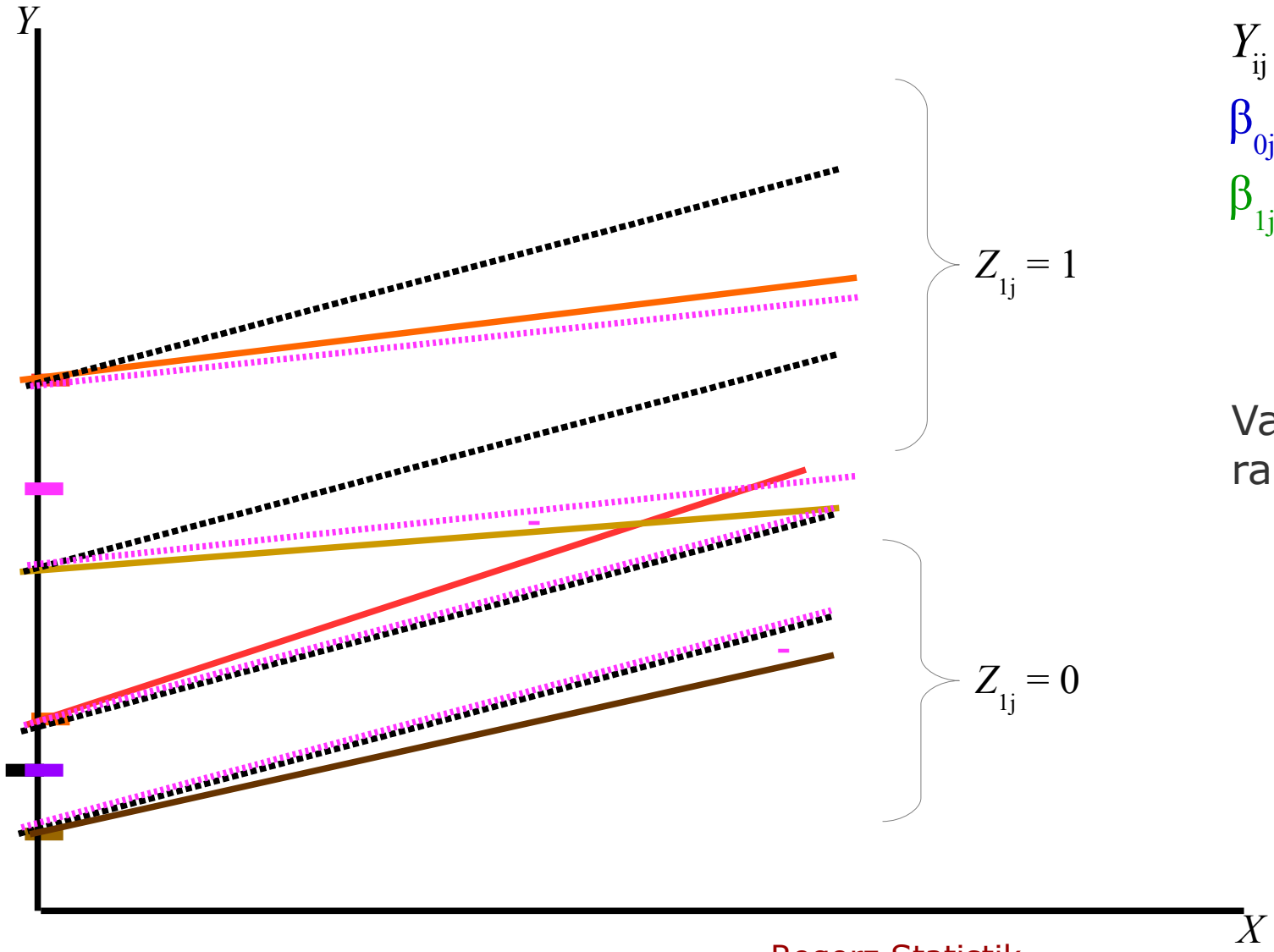
Parameterschätzungen
fixed Effects



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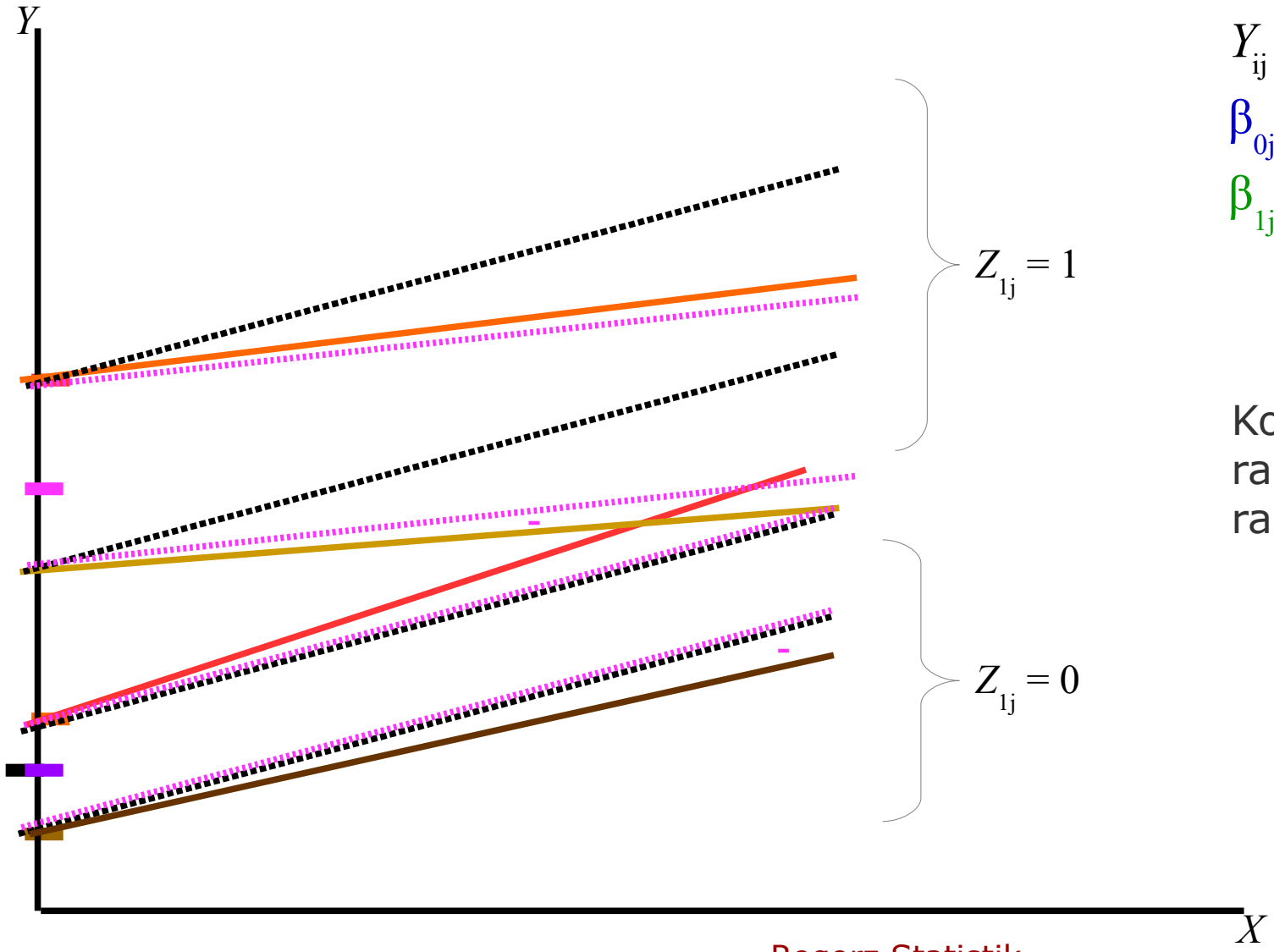
Varianzschätzungen
random Effects



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$$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + u_{1j} \quad \text{(L2)}$$

Kovarianzschätzung random intercept mit random slope

Nötige Ergänzungen

- Stichprobengröße (insbes. L2)
- Regressionsvoraussetzungen
- Zentrierung wg. Interpretation

Hilfreich?

www.regorz-statistik.de

Impressum:

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mail@regorz-statistik.de

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